2016 Higher Maths Paper 2

1 a) $M(2,4)$ $\quad m_{PM} = \frac{8}{2}$

$= 4$

PM is $y = 4x - 4$

b) $m_{PR} = 1 \quad \therefore m_{L} = -1$

$y - 4 = -1(x - 2)$

$y - 4 = -x + 2$

$y = -x + 6$

c) Midpoint PR $(5,1)$

$y = -x + 6$ PR on line L

$1 = -5 + 6$

$1 = 1$ ✓

2. for no real roots $b^2 - 4ac < 0$

$(-2)^2 - 4(1)(3 - p) < 0$

$4 - 12 + 4p < 0$

$-8 + 4p < 0$

$4p < 8$

$p < 2$

3 a)

-1	2	-9	3	14
	0	-2	11	-14
	2	-11	14	0

no remainder, so $(x+1)$ is a factor

$2x^3 - 9x^2 + 3x + 14 = 0$

$(x+1)(2x^2 - 11x + 14) = 0$

$(x+1)(2x-7)(x-2) = 0$

$x = -1, \quad x = 2, \quad x = 7/2$

b) $$\int_{-1}^{2} 2x^3 - 9x^2 + 3x + 14 \, dx$$

$$= \left[\frac{1}{2}x^4 - 3x^3 + \frac{3}{2}x^2 + 14x \right]_{-1}^{2}$$

$$= \left(\frac{1}{2}(2)^4 - 3(2)^3 + \frac{3}{2}(2)^2 + 14(2) \right) - \left(\frac{1}{2}(-1)^4 - 3(-1)^3 + \frac{3}{2}(-1)^2 + 14(-1) \right)$$

$$= (18) - (-9)$$

$$= 27 \text{ units}^2$$

4 Circle 1 Centre $(-5, 6)$ radius 3

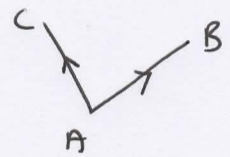
Circle 2 centre $(3, 0)$ radius 5

distance between centres is $\sqrt{8^2 + 6^2} = \sqrt{100}$

$= 10$

Since $r_1 + r_2 <$ distance between circles

there is no intersection.

5. 

$\vec{AB} = \begin{pmatrix} -8 \\ 16 \\ 2 \end{pmatrix}$ $\vec{AC} = \begin{pmatrix} -2 \\ -8 \\ 16 \end{pmatrix}$

$$|\vec{AB}| = \sqrt{(-8)^2 + 16^2 + 2^2}$$
$$= 18$$

$$|\vec{AC}| = \sqrt{(-2)^2 + (-8)^2 + 16^2}$$
$$= 18$$
$$= |\vec{AB}|$$

$$\vec{AB} \cdot \vec{AC} = -8 \times -2 + 16 \times -8 + 2 \times 16$$
$$= -80$$

$$\text{Cos BAC} = \frac{\vec{AB} \cdot \vec{AC}}{|\vec{AB}||\vec{AC}|}$$

$$= \frac{-80}{324}$$

$\underline{\text{BAC} = 104.29^\circ}$

6. $B(t) = 200e^{0.107t}$

a) $t = 0$ $B(t) = 200$

b) $400 = 200e^{0.107t}$
$2 = e^{0.107t}$
$\ln 2 = \ln e^{0.107t}$
$\ln 2 = 0.107t$
$\frac{\ln 2}{0.107} = t$

$\underline{t = 6.48 \text{ hours}}$

7. a) Area $= 6xy$

$108 = 6xy$

$\frac{18}{x} = y$

$y = \frac{18}{x}$

$P = 9x + 8y$

$P = 9x + \frac{144}{x}$ as required.

b) for minimum length $L'(x) = 0$

$L(x) = 9x + 144x^{-1}$ $\quad L'(x) = 9 - 144x^{-2}$

$9 - \frac{144}{x^2} = 0$

$9 = \frac{144}{x^2}$

$x^2 = 16$

$x = \pm 4$

$x \neq -4$

x	$\rightarrow$	4	$\rightarrow$
	$+$	0	$-$
$L'(x)$	\	—	/

minimum valve at $x = 4$ metres.

8 a) $k\cos(x+a) = k\cos x\cos a - k\sin x\sin a$

$5\cos x \quad - 2\sin x$

$k\cos a = 5 \qquad k = \sqrt{29} \qquad \tan a = \frac{2}{5} \qquad a = 0.381$

$k\sin a = 2$

$5\cos x - 2\sin x = \sqrt{29}\cos(x + 0.381)$

b) $10 + 5\cos x - 2\sin x = 12$

$5\cos x - 2\sin x = 2$

$\sqrt{29}\cos(x + 0.381) = 2$

$\cos(x + 0.381) = 2/\sqrt{29}$

$x + 0.381 = 1.19 \qquad x + 0.381 = 5.09$

$x = 0.809 \qquad x = 4.709$

$P(\underline{0.809}, 12) \qquad Q(\underline{4.709}, 12)$

9. $f(x) = \int \frac{2x+1}{\sqrt{x}}\,dx$

$$\int \frac{2x+1}{\sqrt{x}}\,dx = \int 2\sqrt{x} + \frac{1}{\sqrt{x}}\,dx$$

$$= \int 2x^{1/2} + x^{-1/2}\,dx$$

$$f(x) = \frac{4}{3}x^{3/2} + 2x^{1/2} + C$$

If $f(9) = 40$

$$40 = \frac{4}{3}(9)^{3/2} + 2(9)^{1/2} + C$$

$$40 = 36 + 6 + C$$

$$40 = 42 + C$$

$$C = -2$$

$$f(x) = \frac{4}{3}x^{3/2} + 2x^{1/2} - 2$$

or.

$$f(x) = \frac{4}{3}\sqrt{x^3} + 2\sqrt{x} - 2$$

10. $y = (x^2+7)^{1/2}$ $\quad \frac{dy}{dx} = \frac{1}{2}(x^2+7)^{-1/2} \times 2x$

$$= x(x^2+7)^{-1/2}$$

$$= \frac{x}{\sqrt{x^2+7}}$$

b) $\int \frac{4x}{\sqrt{x^2+7}}\,dx = 4\int \frac{x}{\sqrt{x^2+7}}\,dx$

$$= 4\left((x^2+7)^{1/2}\right) + C$$

$$= 4(x^2+7)^{1/2} + C$$

11 a) Given that $\sin 2x = 2\sin x \cos x$ and $\tan x = \frac{\sin x}{\cos x}$

and $\cos 2x = 1 - 2\sin^2 x$

so $2\sin^2 x = 1 - \cos 2x$

$$\sin 2x \tan x = \sin 2x \frac{\sin x}{\cos x}$$

$$= 2\sin x \cos x \frac{\sin x}{\cos x}$$

$$= 2\sin^2 x$$

$$= 1 - \cos 2x \quad \text{as required}$$

b) $f(x) = \sin 2x \tan x$

$= 1 - \cos 2x$ from (a)

$f'(x) = 0 - -2\sin 2x$

$= \underline{2\sin 2x}$